

Introduction to Stochastic Cooling

2nd September 2015

Hans Stockhorst

Forschungszentrum Jülich GmbH, Institut für Kernphysik/COSY

BND School 2015

Landal Resort Eifeler Hof, Heimbach

August 31th – September 11th, 2015

Aim of Beam Cooling

- Reduce transverse (horizontal and vertical) beam dimensions
- Reduce momentum spread in the beam
- No beam losses: *phase space density increase*

- *Beam accumulation of (rare) particles:*
Cooling to make space available so that more particles can be stacked in the storage ring
- *Internal Target Experiments:*
Compensate growth in beam size,
compensate mean energy loss,
keep good beam-target overlap,
keep momentum resolution

Early History of Stochastic Cooling at CERN

1968 Idea of stochastic cooling by van der Meer (CERN)

1972 First observation of Schottky noise at ISR.

1972 Theory of transverse stochastic cooling.

1975 First experimental demonstration of transverse cooling at ISR.

1975 pbar accumulation schemes for ISR and SPS.

1975 Filter method for momentum cooling. Palmer (BNL), Thorndahl (CERN)

1977 & 1978 Refinement of theory and detailed experimental verification at ICE.

1981 & 1982 Accumulation of several 10^{11} pbars in AA from batches of several 10^6 pbars.

1986 & 1987 Construction of AC.

1983 Observation of W and Z bosons (carriers of the electro-weak force).

1995 Observation of top-quark at Fermilab.

S. van der Meer (1925 – 2011): Nobel prize in physics 1984

D. Möhl (1936 -2012): pioneer in stochastic cooling theory and beam physics

F. Caspers and D. Möhl, “History of stochastic beam cooling and its application in many different projects”,
Eur.Phys.J. H36 (2012) 601-632

Description of Stochastic Cooling

■ Time Domain: Sampling Picture

- Statistical approach
- Yields cooling rate equation
- Mixing, electronic and particle noise introduced heuristically

■ Frequency Domain:

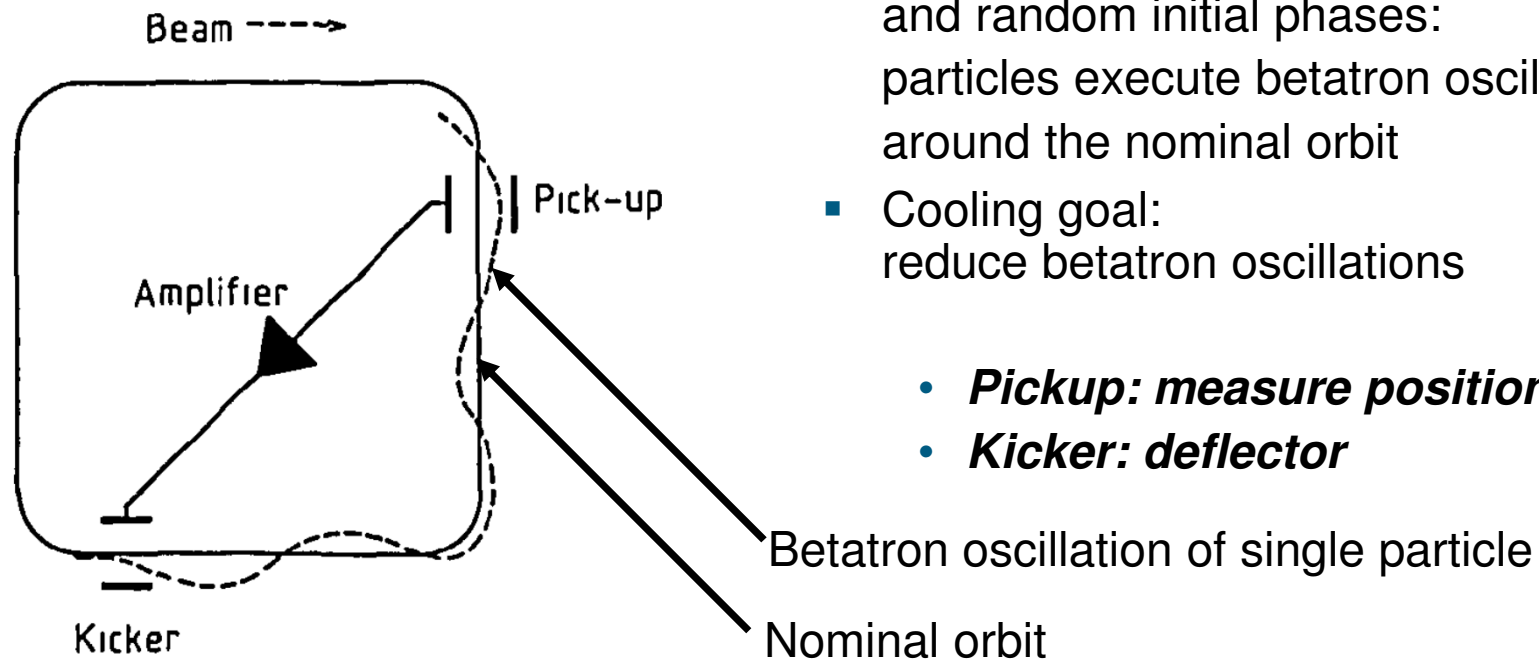
- Yields precise description of cooling with Mixing
- Description of Betatron Cooling
- Description of Momentum Cooling:
 - *Filter Method, Palmer Cooling, Time of Flight Cooling*
- Hardware can be included in description
(PU and KI response, amplifiers, Filters, power etc.)

References: D. Möhl, CERN Accelerator School CAS, CERN 87-03, 1987

D. Möhl, Stochastic Cooling of Particle Beams, Lecture Notes in Physics 866, Springer 2013

Time Domain Description of Betatron Cooling

Basic Setup:



Courtesy by D. Möhl, CERN

Electronic delay adjusted to particle travelling time from PU to Ki for nominal particle ($\Delta p/p = 0$)

Betatron Oscillations

Smooth sinusoidal approximation $\beta \approx R/Q$

betatron oscillation $x(s)$:

$$x(s) = a \cdot \cos(\psi(s) + \varphi)$$

$$y(s) = \frac{R}{Q} \frac{dx(s)}{ds} = -a \cdot \sin(\psi(s) + \varphi)$$

s : position along the ring

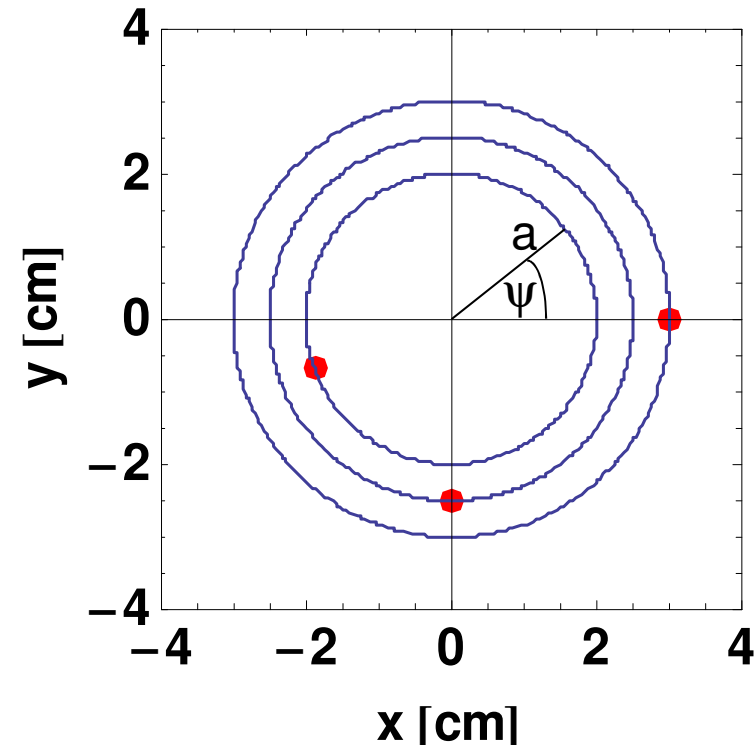
R : ring radius

φ : initial phase

Betatron phase: $\psi(s) = Q \cdot s/R$

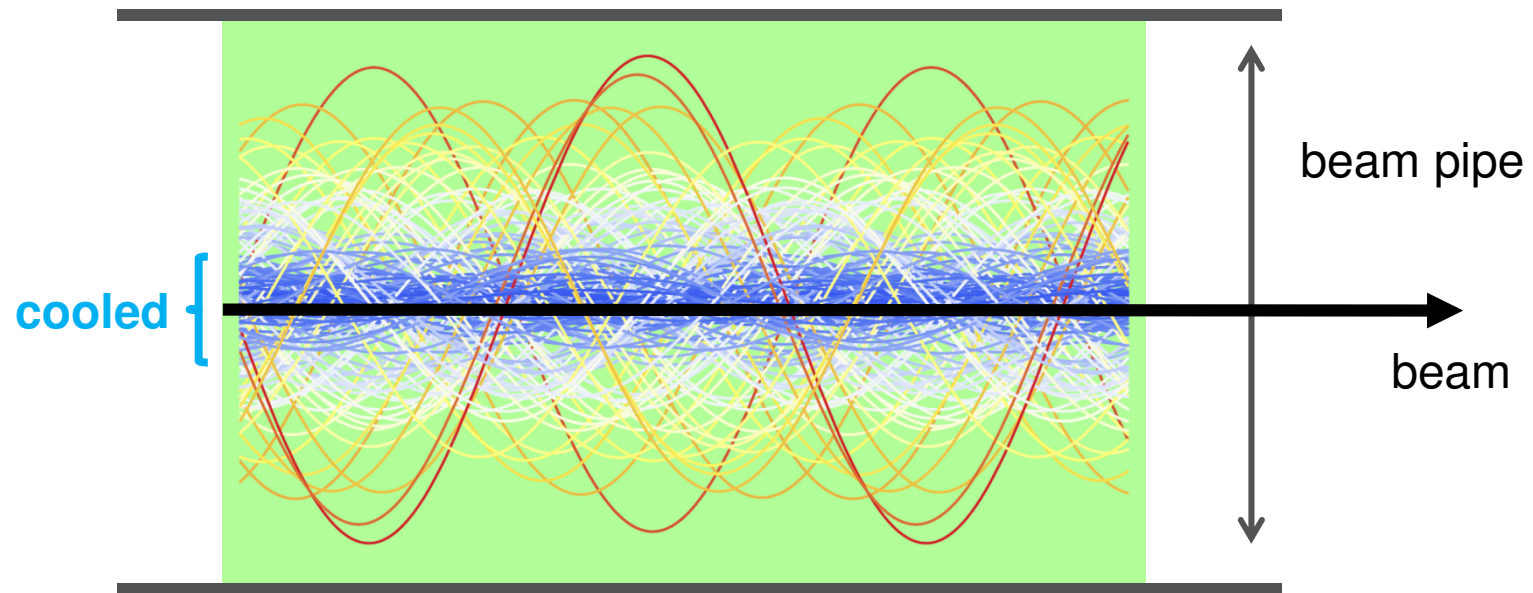
- Q : number of betatron oscillations per turn
- **Q is no rational number (therefore: neither an integer nor an half integer)**

Normalized Phase Space
at position s in the ring:



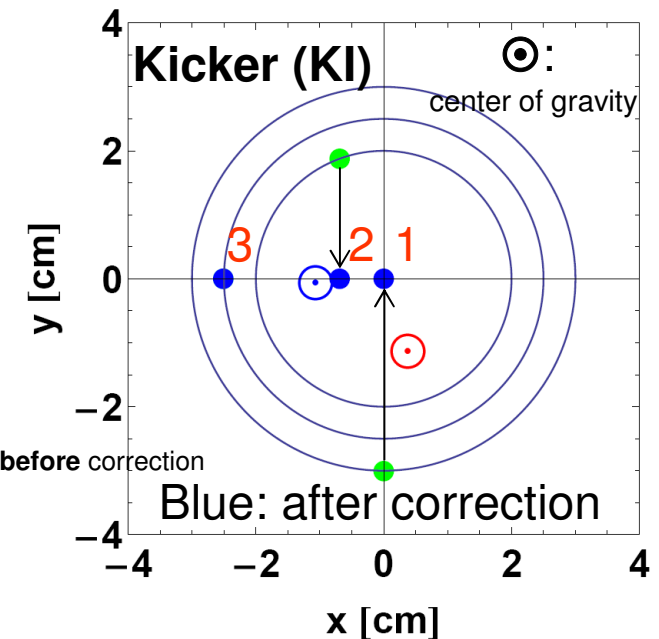
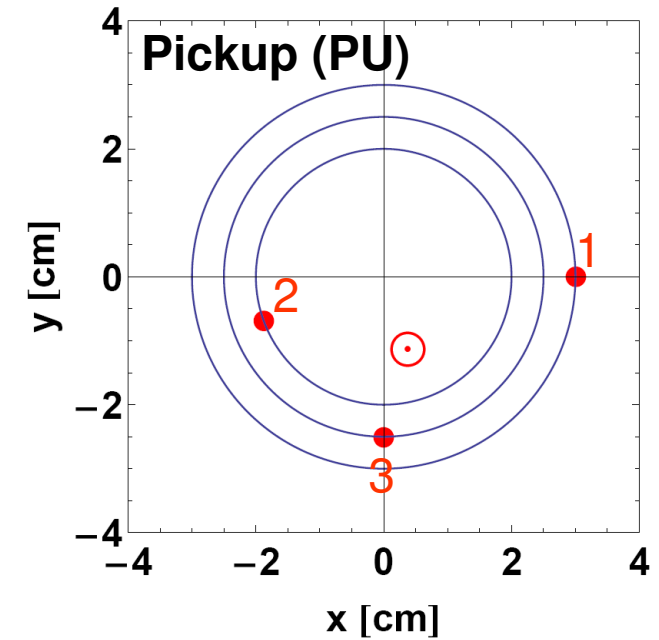
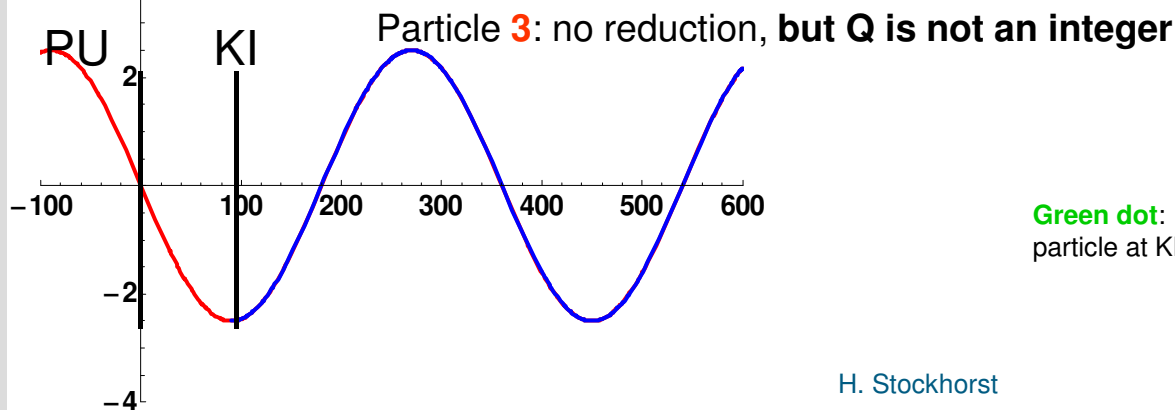
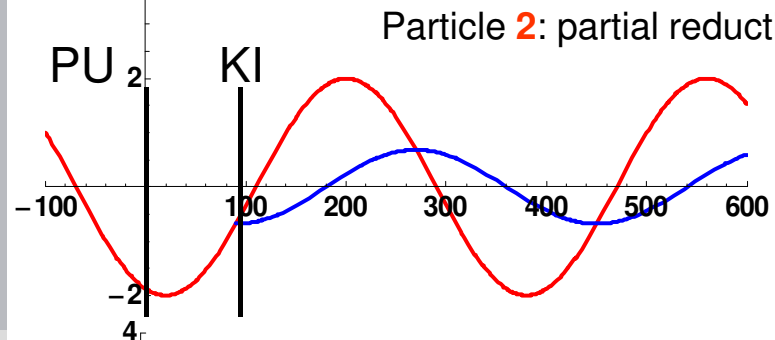
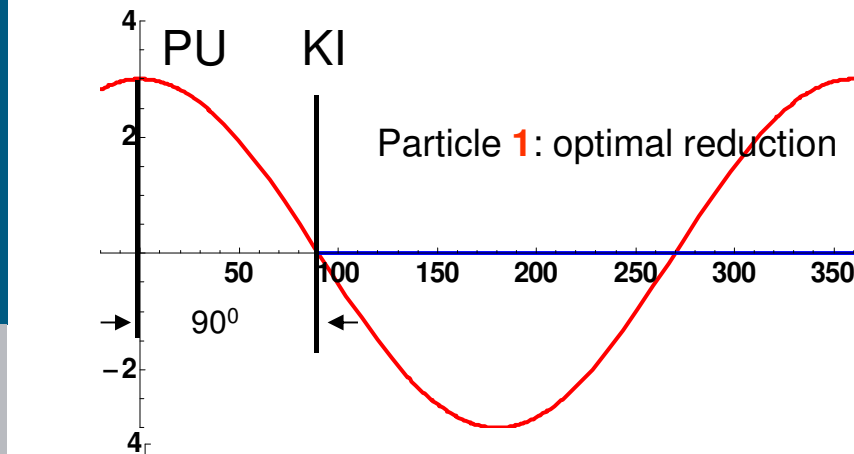
Beam emittance $\varepsilon \sim a^2$

Betatron Motion (artist's view)



- Goal for betatron cooling: reduce amplitude of betatron oscillation

Optimal phase advance $\Delta\psi$ from PU to KI: 90 degrees plus (even) multiples of 180 degrees



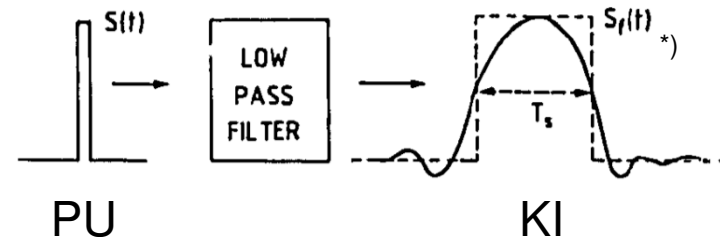
Green dot:
particle at KI before correction

Finite Cooling Bandwidth W

- Single particle can not be resolved
- Finite bandwidth W of the cooling system:**
 - A single particle produces a pulse at the pickup which is broadened to T_s due to the finite bandwidth

Sample time:

$$T_s = \frac{1}{2W}$$



- A particle at time t_0 at the pickup is not only kicked due to its own error signal (**coherent effect**) but also due to other particles in the time interval $t_0 - T_s/2 \leq t \leq t_0 + T_s/2$:
 - the finite bandwidth results in **HEATING (incoherent effect)** due to the other particles in the sample of length T_s

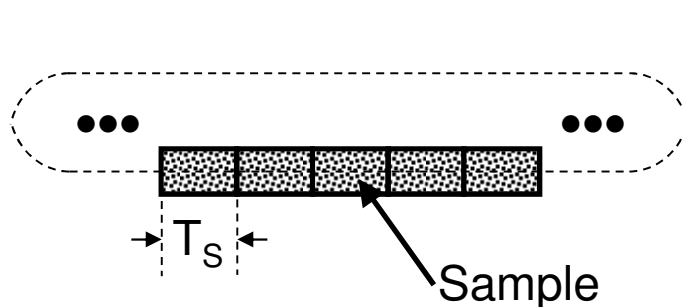
single test particle picture ➡ sampling picture of cooling

Beam Samples

- Finite bandwidth W of the cooling loop:
 - The cooling system resolves samples of the beam***

In a **continuous coasting beam (DC beam)** of length T (revolution period) and particle number N :

- Number of equally spaced samples:



$$\ell_s = \frac{T}{T_s} = 2WT$$

- Number of particles per sample: $N_s = \frac{N}{\ell_s} = \frac{N}{2WT}$
(sample size)

Example:

$$N = 10^{10}, T = 1 \mu\text{s}, W = 2 \text{ GHz}$$

$$T_s = 250 \text{ ps}$$

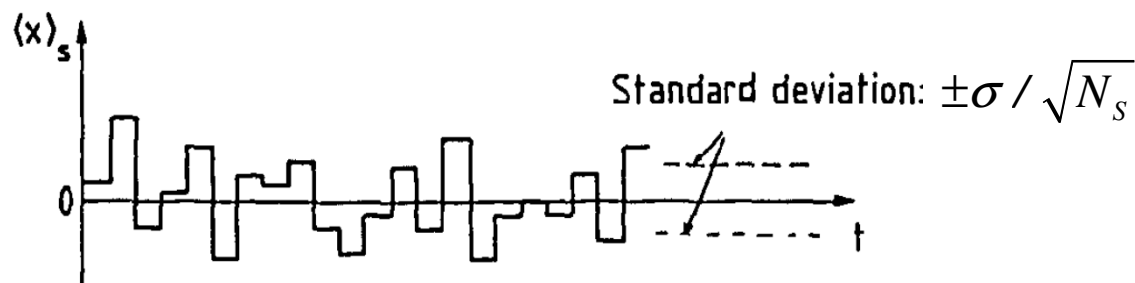
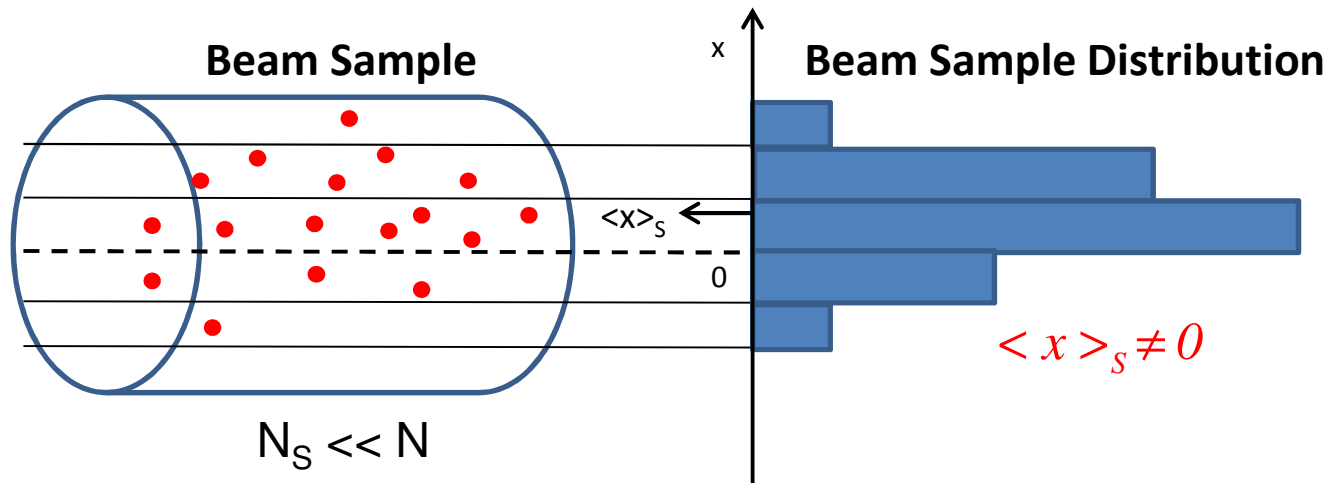
$$\ell_s = 4000$$

$$N_s = 2.5 \cdot 10^6$$

Beam Sample

Take a beam with N particles

with mean $E[x] = 0$ and variance $\sigma^2 = E[x^2] \neq 0$

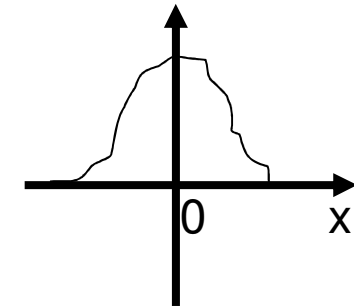


- Sample averages fluctuate around zero
- Standard deviation $\pm \sigma / \sqrt{N_s}$

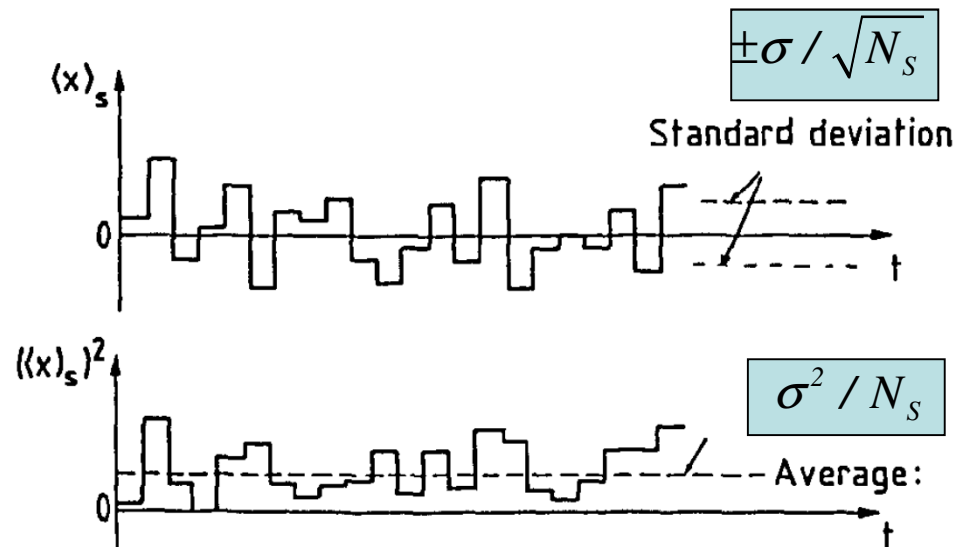
Sample Statistics

Take a beam with N particles

with mean $E[x] = 0$ and variance $\sigma^2 = E[x^2] \neq 0$



- The sample averages $\langle x \rangle_s$ will fluctuate around zero:



Courtesy by D. Möhl, CERN

Sample Relations:

Average over the samples:

$$\text{Mean: } E[\langle x \rangle_s] = E[x] = 0$$

$$\text{Variance: } E[\langle x^2 \rangle_s] = \sigma^2$$

$$\text{Squared mean: } E[(\langle x \rangle_s)^2] = \frac{\sigma^2}{N_s}$$

$$\mu = E[x] = \int x \Psi(x) dx \quad \sigma^2 = E[x^2] = \int (x - \mu)^2 \Psi(x) dx \quad 1 = \int \Psi(x) dx$$

Example:

- Discrete distribution with values $x = -1, 0, 1$ having equal probability:
 $\mu = E[x] = 0$ and $\sigma^2 = E[x^2] = 1/3 ((-1)^2 + 0^2 + 1^2) = 2/3$
- Form all possible samples with size $N_S = 2$

Before Correction:			
Sequence:		Sample average $\langle x \rangle_S$	variance $\langle x^2 \rangle_S$
Sample	-1 -1	-1	1
▪	-1 0	-1/2	1/2
▪	-1 1	0	1
▪	0 -1	-1/2	1/2
▪	0 0	0	0
▪	0 1	1/2	1/2
▪	1 -1	0	1
▪	1 0	1/2	1/2
▪	1 1	1	1
mean:		0	
variance:			2/3
(average over 9 samples)			

$$\langle x \rangle_S = \frac{1}{N_S} \sum_{k=1}^{N_S} x_k$$

$$\langle x^2 \rangle_S = \frac{1}{N_S} \sum_{k=1}^{N_S} x_k^2$$

$$E[\langle x \rangle_S] = \frac{1}{9} \sum_{k=1}^9 [\langle x \rangle_S]_k = E[x] = 0$$

$$E[\langle x^2 \rangle_S] = \frac{1}{9} \sum_{k=1}^9 [\langle x^2 \rangle_S]_k = \sigma^2$$

Towards the Cooling Equation

The test particle receives a correction proportional to its error x : $\Delta x = \lambda \cdot x$

The error x will change from x to x_C : $x_C = x - \lambda x$

Finite bandwidth \Leftrightarrow finite sample size:

$$x_C = x - \lambda x - \lambda \sum_{\text{others}} x_i$$

Others:
all particles in the sample
except the test particle

↑
Coherent
cooling

↘
Incoherent, heating

Or: $x_C = x - (\lambda N_s) \frac{1}{N_s} \sum_{\text{sample}} x_i \Rightarrow x_C = x - g \cdot \langle x \rangle_s$

- The test particle receives a correction proportional to the sample average $\langle x \rangle_s$

Application of Sample Relations

Sample the beam: **mean** $\langle x \rangle_S = \frac{1}{N_S} \sum_{i=1}^{N_S} x_i$ **variance** $\langle x^2 \rangle_S = \frac{1}{N_S} \sum_{i=1}^{N_S} x_i^2$

1. First correction: Each particle in the sample is corrected as $x_k \rightarrow x_k - g \langle x \rangle_S$

2.a Calculate new sample variance: $\langle x^2 \rangle_S^{new} = \frac{1}{N_S} \sum_{k=1}^{N_S} (x_k - g \langle x \rangle_S)^2$

2.b Calculate new sample mean: $\langle x \rangle_S^{new} = \frac{1}{N_S} \sum_{k=1}^{N_S} (x_k - g \langle x \rangle_S) = (1 - g) \langle x \rangle_S$!

3. Repeat 1. & 2. for all samples and average over all samples:

Apply sample relations: $\sigma_C^2 = \sigma^2 - \frac{1}{N_S} (2g - g^2) \cdot \sigma^2$ $\sigma_C^2 = E \left[\frac{1}{N_S} \sum_{k=1}^{N_S} (x_k - g \langle x \rangle_S)^2 \right]$

Change of beam variance in **one turn**: $\Delta \sigma^2 = \sigma_C^2 - \sigma^2 = -\frac{1}{N_S} (2g - g^2) \cdot \sigma^2$

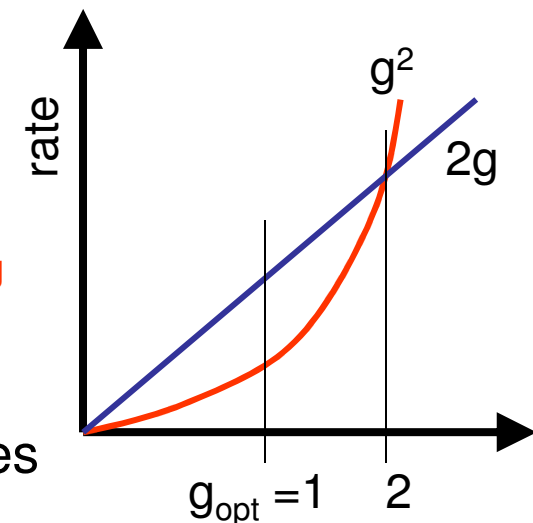
Cooling Rate Equation (simple form)

Since $T_s = \frac{I}{2W} \Rightarrow$ number of particles in a sample: $N_s = \frac{N}{T} \frac{I}{2W}$

$$\frac{I}{\tau_{\sigma^2}} = -\frac{I}{\sigma^2} \frac{d\sigma^2}{dt} = -\frac{I}{T} \frac{\Delta\sigma^2}{\sigma^2} = \frac{2W}{N} (2g - g^2)$$

↑
↑
heating

cooling



- Use large bandwidth W
- Large particle number $N \Rightarrow$ cooling rate decreases
- Choose optimum gain $g = 1$
- **Optimal cooling rate**

$$\left(\frac{I}{\tau_{\sigma^2}} \right)_{opt} = \frac{2W}{N}$$

Example: Simulation of One-Turn Correction

- Discrete distribution with values $x = -1, 0, 1$ having equal probability:
 $\mu = E[x] = 0$ and $\sigma^2 = E[x^2] = 1/3 ((-1)^2 + 0^2 + 1^2) = 2/3$
- Form all possible samples with $N_S = 2$ **g = 1**

Before Correction:				After Correction:			
Sequence:		Sample average <x>S	variance <x^2>S	Sequence:		Sample average	variance
-1	-1	-1	1	0	0	0	0
-1	0	- 1/2	1/2	- 1/2	1/2	0	1/4
-1	1	0	1	-1	1	0	1
0	-1	- 1/2	1/2	1/2	- 1/2	0	1/4
0	0	0	0	0	0	0	0
0	1	1/2	1/2	- 1/2	1/2	0	1/4
1	-1	0	1	1	-1	0	1
1	0	1/2	1/2	1/2	- 1/2	0	1/4
1	1	1	1	0	0	0	0
mean:		0		mean:		0	
variance:			2/3	variance:			1/3
(average over 9 samples)							

$$\langle x \rangle_S = \frac{1}{N_S} \sum_{k=1}^{N_S} x_k$$

$$\langle x^2 \rangle_S = \frac{1}{N_S} \sum_{k=1}^{N_S} x_k^2$$

One turn:

$$\sigma^2 = 2/3 \rightarrow \sigma^2 = 1/3$$

Example: Mixing and second correction

Before Correction:				After Correction:			
		after Mixing					
Sequence:		Sample average	variance	Sequence:		Sample average	variance
		$\langle x \rangle_S$	$\langle x^2 \rangle_S$				
0	1/2	1/4	1/8	- 1/4	1/4	0	1/16
- 1/2	1	1/4	5/8	- 3/4	3/4	0	9/16
-1	- 1/2	- 3/4	5/8	- 1/4	1/4	0	1/16
1/2	0	1/4	1/8	1/4	- 1/4	0	1/16
0	1/2	1/4	1/8	- 1/4	1/4	0	1/16
- 1/2	-1	- 3/4	5/8	1/4	- 1/4	0	1/16
1	- 1/2	1/4	5/8	3/4	- 3/4	0	9/16
1/2	0	1/4	1/8	1/4	- 1/4	0	1/16
0	0	0	0	0	0	0	0
mean:		0		mean:		0	
variance:			1/3	variance:			1/6
(average over 9 samples)							

- After mixing:
sample average reappears!
- Cooling!

First turn:

Second turn:

$$\sigma^2 = 2/3 \rightarrow \sigma^2 = 1/3 \rightarrow \sigma^2 = 1/6$$

Mixing

- In the model up to now:
After one turn **sample averages are zero**
⇒ error signals vanish: cooling will stop!?
- Fortunately:
Particles in the beam have different momenta: $\Delta p \neq 0$
⇒ spread in revolution time ΔT
⇒ **particles will mix: sample error reappears**
⇒ cooling proceeds until all particles have zero error

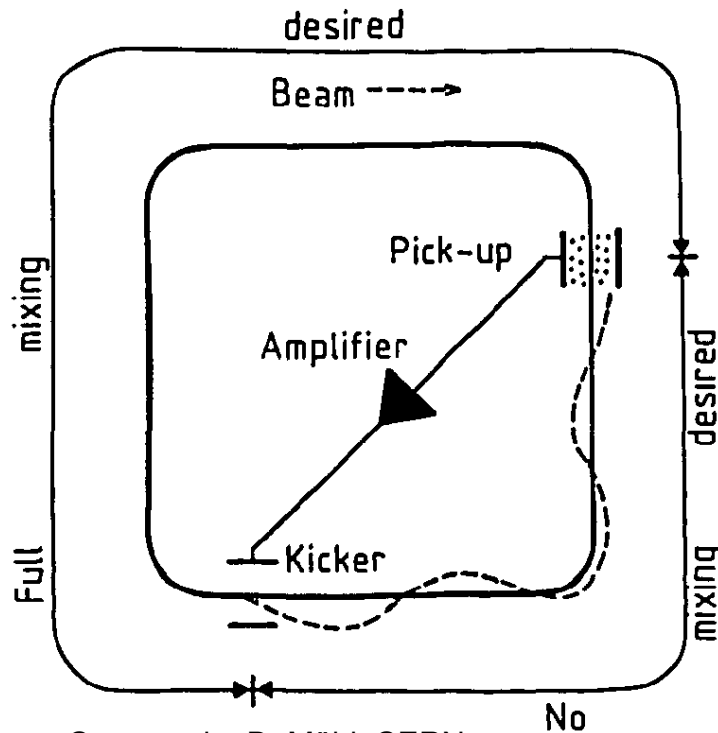
For one revolution: $\frac{\Delta T}{T} = -\eta \frac{\Delta p}{p} \quad \eta = \frac{l}{\gamma^2} - \alpha \quad \alpha = \frac{1}{L} \int_0^L \frac{D(s)}{\rho(s)} ds$

$\gamma = E/m_0 c^2$ $D(s)$: dispersion ρ : radius of curvature L : ring length

Mixing factor M: number of turns a particle with ΔT needs to migrate by one sample length T_s :

$$M = \frac{T_s}{|\Delta T|} = \frac{l}{2WT |\eta \Delta p / p|}$$

The Mixing Dilemma



Courtesy by D. Möhl, CERN

Delay for signal from PU to KI set to nominal particle

Kicker to Pickup:

$$\frac{\Delta T_{KP}}{T_{KP}} = -\eta_{KP} \frac{\Delta p}{p} \quad \eta_{KP} = \frac{1}{\gamma^2} - \alpha_{KP} \quad \alpha_{KP} = \frac{1}{s_{KP}} \int_{s_{KP}} \frac{D(s)}{\rho(s)} ds$$

Pickup to Kicker:

$$\frac{\Delta T_{PK}}{T_{PK}} = -\eta_{PK} \frac{\Delta p}{p} \quad \eta_{PK} = \frac{1}{\gamma^2} - \alpha_{PK} \quad \alpha_{PK} = \frac{1}{s_{PK}} \int_{s_{PK}} \frac{D(s)}{\rho(s)} ds$$

Mixing factor PU to KI: $M^* = \frac{T_S/2}{|\Delta T_{PK}|}$

- M should be small, best value M = 1 (good mixing)
- M* should be large (no mixing)

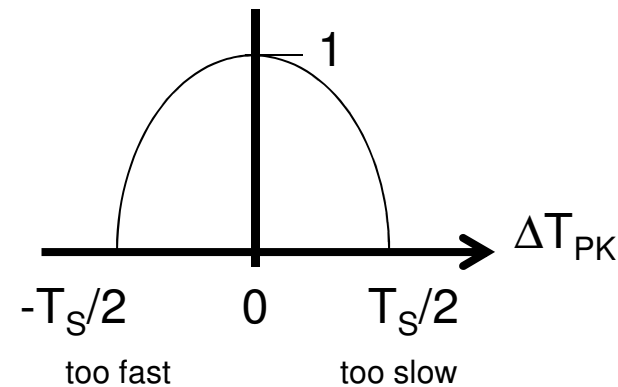
Unwanted Mixing from PU to KI:

- Correction of coherent term -

Kicker pulse:

$$K(\Delta T_{PK}) = 1 - \left(\frac{\Delta T_{PK}}{T_S/2} \right)^2 = 1 - \frac{1}{M^{*2}}$$

$$M^* = \frac{T_S/2}{\Delta T_{PK}}$$



Time at kicker: $T_{PK} = T_{ref} + \Delta T_{PK}$

- Nominal particle ($\Delta p/p = 0$) receives maximum correction $K = 1$.
- Particles which are too slow or too fast receives a reduced correction $K < 1$.

Improved Cooling Rate for betatron motion $x(s)$, $\sigma = x_{\text{rms}}$

$$\frac{1}{\tau_{\sigma^2}} = -\frac{1}{\sigma^2} \frac{d\sigma^2}{dt} = \frac{2W}{N} \cdot \left\{ \underbrace{2g \cdot \sin(\Delta\mu)}_{\text{Unwanted mixing}} \cdot \underbrace{\left(1 - \frac{1}{M^{*2}}\right)}_{\text{Wanted mixing}} - \underbrace{g^2 (M + U)}_{\text{Electronic noise}} \right\}$$

- **Unwanted** mixing from PU to KI:

M^* should be large:
no mixing from PU to Ki

- For Betatron cooling:
 $\Delta\mu$: phase advance from PU to KI
 - *optimal: $\Delta\mu = \pi/2 \bmod \pi$*

- **Wanted** mixing from KI to PU:

$M \geq 1$ should be small,
best value $M = 1$

- **Electronic noise:**
increase in heating,
additional term $g^2 U$
 - **$U \sim 1/N\sigma^2$:**
noise to signal ratio

Improved Optimal Betatron Cooling Rate

With optimal gain:

$$g_{opt} = \frac{M^+}{M+U} \quad \text{with} \quad M^+ = 1 - \frac{1}{M^{*2}} \quad \text{for } \Delta\mu = \pi/2$$

$$\left(\frac{1}{\tau_{\sigma^2}} \right)_{opt} = - \frac{1}{x_{rms}^2} \frac{dx_{rms}^2}{dt} = \frac{2W}{N} \cdot \frac{(M^+)^2}{M+U}$$

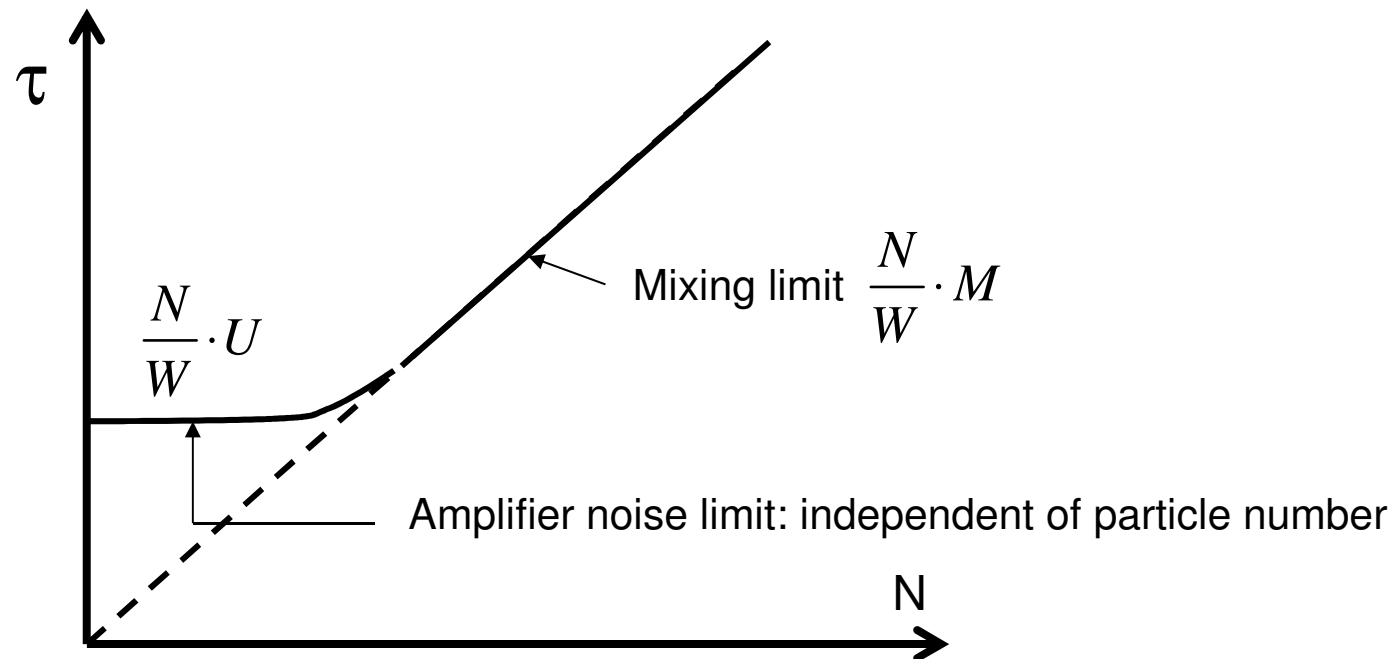
Betatron motion $\sigma^2 = x_{rms}^2$

$$\left(\frac{1}{\tau_{\varepsilon}} \right)_{opt} = - \frac{1}{\varepsilon_{rms}} \frac{d\varepsilon_{rms}}{dt} = \frac{W}{N} \cdot \frac{(M^+)^2}{M+U}$$

Emittance ε

Asymptotic Behavior of Cooling Time τ

$$\tau \sim \frac{N}{W}(M + U) \quad \text{with} \quad U \approx \frac{1}{N}$$



Noise-To-Signal Ratio for Betatron Cooling (1)

From frequency domain approach of cooling:

$$U = \frac{\text{electronic noise power}}{\text{Schottky particle power}} = \frac{k(T_A + T_R)G_A^2 W}{N(Ze)^2 f_0 \frac{|Z'_P|^2}{Z_C} G_A^2 W \epsilon \cdot \beta_P}$$

k: Boltzmann constant ($= 1.38 \times 10^{-24}$ J/K)

T_A : equivalent amplifier noise temperature

T_R : pickup noise temperature

G_A : amplifier gain

Z'_P : pickup impedance [Ω/m]

Z_C : line impedance (50 Ω)

β_P : betatron function at PU

N: particle number

Ze: particle charge

ϵ : beam emittance

f_0 : revolution frequency

Noise-To-Signal Ratio for Betatron Cooling (2)

To reduce the noise-to-signal ratio U

$$U = \frac{k(T_A + T_R)G_A^2 W}{N(Ze)^2 f_0 \frac{|Z'_P|^2}{Z_C} G_A^2 W \varepsilon \cdot \beta_P}$$

- Cool pickup electrodes and use low noise amplifiers
- Use PU structures with large coupling impedance Z'_P
- Use large betatron function at PU
- **During cooling the emittance ε decreases and U increases**
 - If possible, move electrodes during cooling closer to the beam to increase the coupling impedance Z'_P
- If N and Z large: cooling becomes independent of charge number

Noise-To-Signal Ratio: Example

$$W = 1 \text{ GHz}$$

$$T_A: 20 \text{ K} \quad T_R: 20 \text{ K}$$

$$Z'_{pl} = 3 \text{ k}\Omega/\text{m} \quad Z_C: 50 \text{ }\Omega$$

$$\beta_{PU}: 8 \text{ m}$$

$$G_A: 1$$

$$N = 10^9 \text{ protons}$$

$$f_0 = 1 \text{ MHz}$$

$$\varepsilon = 5 \text{ mm mrad}$$

- **Thermal noise power in 1 GHz: 0.06 pW**
(at room temperature $\approx 1 \text{ pW}$)
- **Schottky signal noise power in 1 GHz: 0.2 pW**

$$\Rightarrow \quad U = 0.3 \text{ (} = 5 \text{ at room temperature } 300 \text{ K!)}$$

$$\Rightarrow \quad \textbf{Cryogenic cooling of pickup electrodes essential!}$$

Time Evolution of Beam Emittance

$$\frac{d\varepsilon}{dt} = -\frac{W}{N} (2gM^+ - g^2M) \cdot \varepsilon + g^2 \frac{W}{N} (U\varepsilon)$$

where $U\varepsilon$ is **independent** from ε

- First order differential equation for ε : $\varepsilon(t) = (\varepsilon(0) - \varepsilon_\infty) e^{-t/\tau} + \varepsilon_\infty$

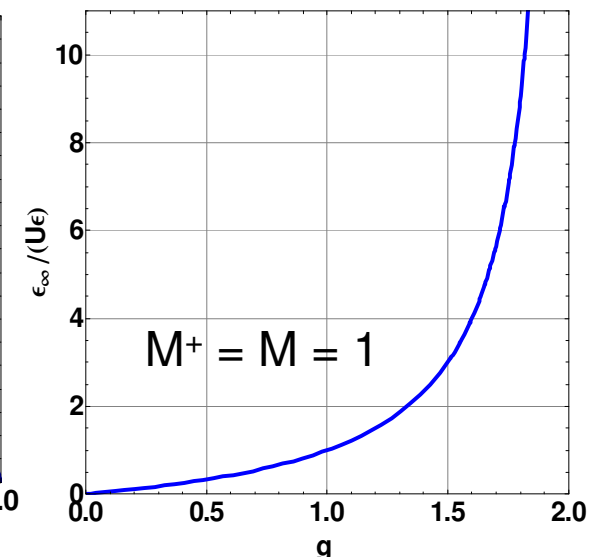
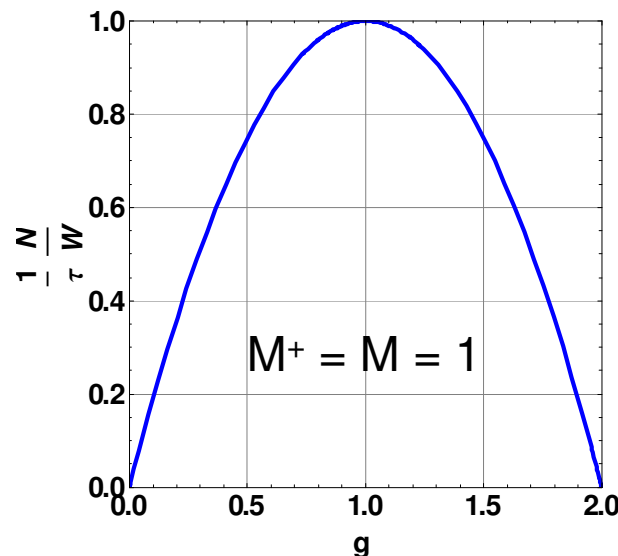
$$\frac{1}{\tau} = \frac{W}{N} (2gM^+ - g^2M)$$

$$\text{Equilibrium emittance: } \varepsilon_\infty = \frac{g}{2M^+ - gM} \cdot (U\varepsilon)$$

Optimal cooling rate:

$$\frac{1}{\tau_{opt}} = \frac{W}{N} \cdot \frac{(M^+)^2}{M}$$

$$\varepsilon_\infty = \frac{1}{M} \cdot \frac{k(T_R + T_A)}{N(Ze)^2 f_0 \frac{|Z'_P|^2}{Z_C} \cdot \beta_P}$$



Stochastic Cooling Systems at COSY (present status)

Stochastic Cooling in COSY:

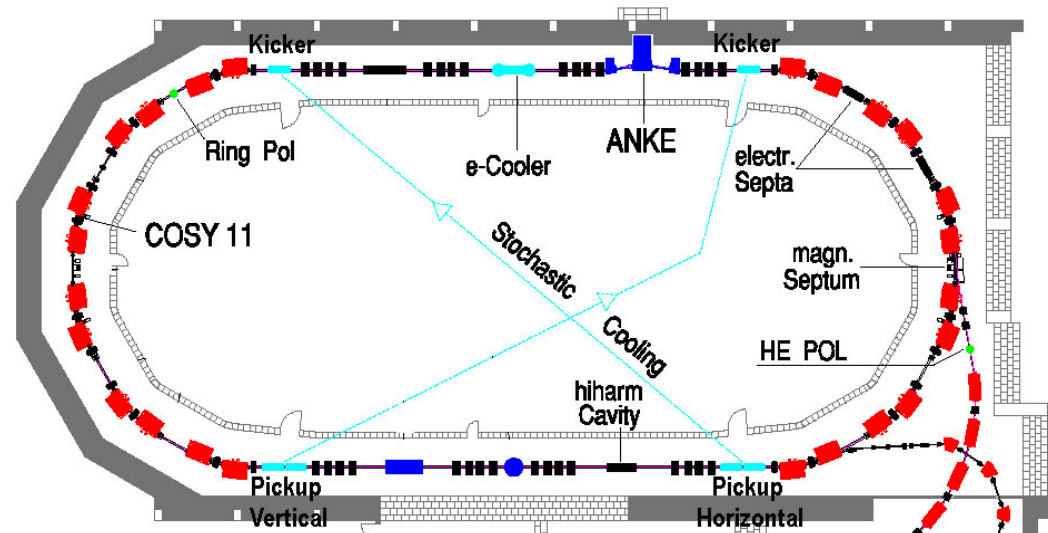
- Horizontal system: 1.8 – 3.0 GHz (band II)
- Vertical system: 1.0 – 3.0 GHz (band I + II)
- Longitudinal system: 1.0 – 1.8 GHz (band I)
- Proton momentum range: 1.5 GeV/c to 3.6 GeV/c
($\beta = v/c > 0.85$)
- **Systems can be independently adjusted**

Longitudinal cooling with **optical notch filter** or **Time-Off-Flight** cooling technique

Example COSY

- Diagonal signal paths:
 $S_{PK} = S_{KP} = L/2$ (L: ring length = 174 m)
- Dispersion zero on straights:
 $\eta_{PK} = \eta$

$$\Rightarrow M^* = M$$



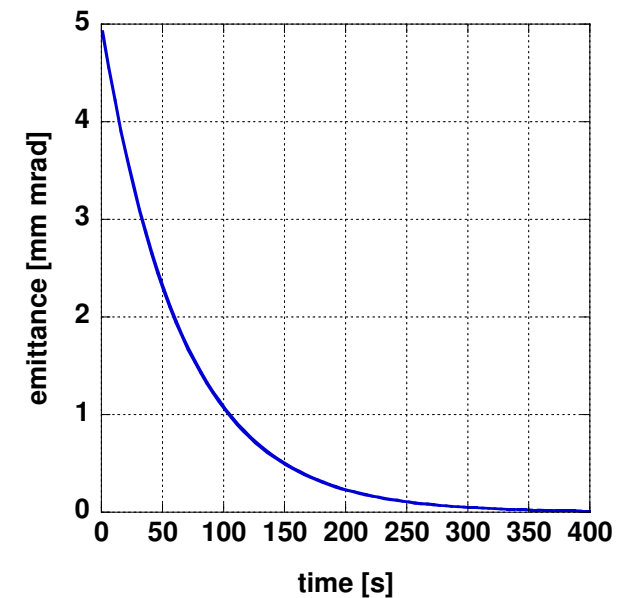
$$W = 2 \text{ GHz} \quad \eta = 0.1$$

$$N = 10^{10} \text{ protons} \quad T = 1 \mu\text{s} \quad \Delta p/p = 2 \cdot 10^{-4}$$

PU structures cooled: $U \approx 0$

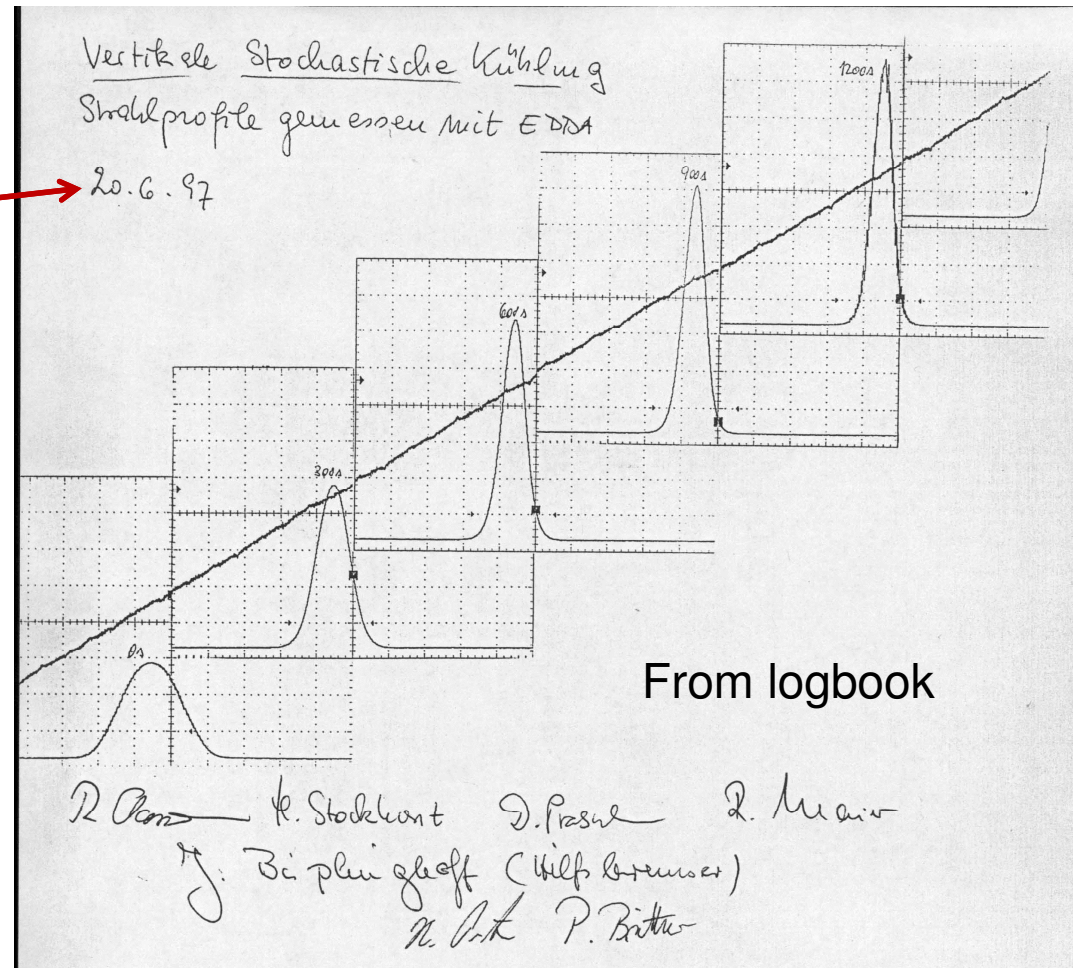
$$\Rightarrow M = \frac{T_s}{|\Delta T|} = \frac{1}{2WT|\eta\Delta p/p|} = 13 \quad \Rightarrow M^* = 13$$

$$\left(\frac{1}{\tau_\varepsilon} \right)_{opt} = \frac{W}{N} \cdot \frac{1}{M} \approx 0.015 \text{ s}^{-1} \quad \text{or} \quad (\tau_\varepsilon)_{opt} \approx 65 \text{ s}$$



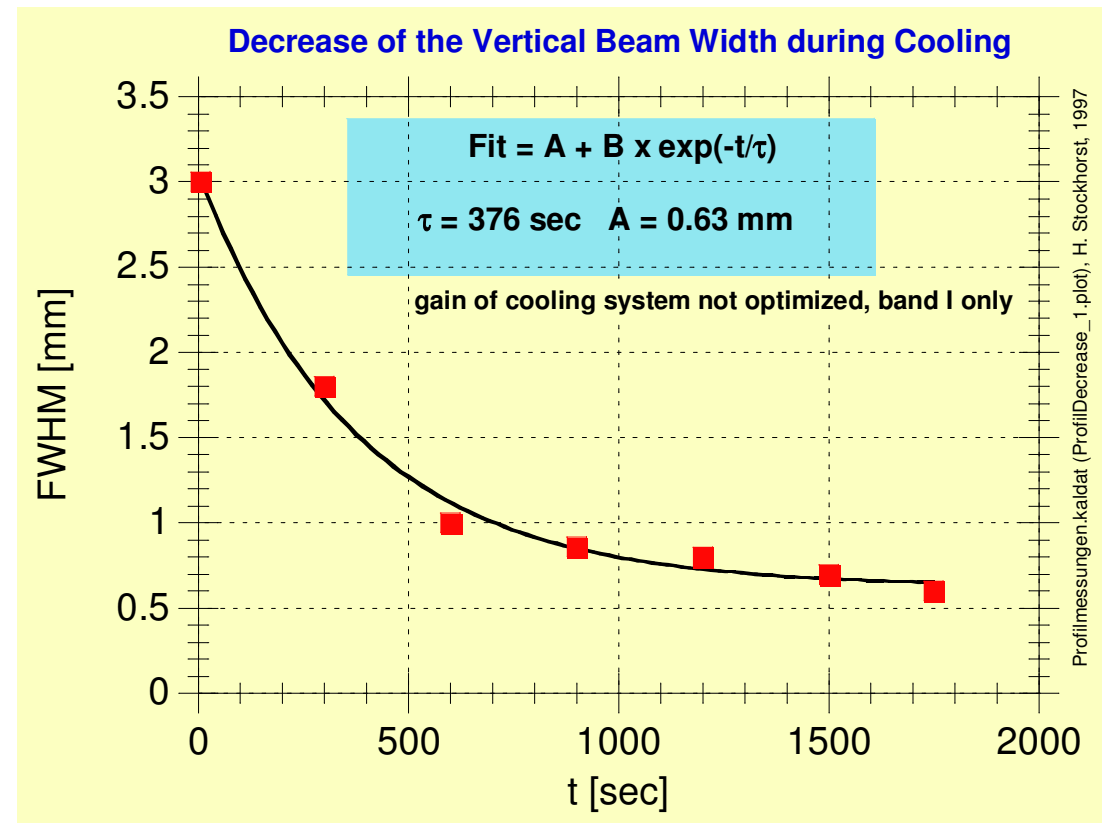
First Successful Betatron Cooling at COSY (1)

- First observation of **vertical** stochastic cooling at COSY eighteen years ago.
- Vertical beam profiles were measured with EDDA.
- Cooling with only band I (1 -1.8) GHz
- Gain and phase not optimized at that time
- No beam losses



First Successful Betatron Cooling at COSY (2)

- The vertical beam width shrinks exponentially towards an equilibrium.
- The FWHM of the beam is reduced by a factor of five after about 1500 s.
- The system was not optimized.



Summary Betatron Cooling

- The cooling rate is proportional to bandwidth W and inversely proportional particle number N
- For stochastic cooling a DC beam is favorable
- Betatron cooling:
 - *Phase advance $\Delta\mu = \pi/2$ plus multiple of π*
 - *Betatron tune Q not an integer or half integer* ✓
- Signal delay adjustment
- Mixing is necessary ($\eta \neq 0$) and is produced by the dispersion in the magnetic lattice
- Mixing between KI and PU should be large, between PU and KI it should be low!
- Mixing is always a compromise, except when the lattice allows to change the optics to make $\eta_{PK} = 0$ and $\eta_{KP} \neq 0$

Stochastic Momentum Cooling

- Approximately: Similar rate equation for the rms relative momentum spread $\sigma = (\Delta p/p)_{rms}$:

$$\frac{1}{\tau_{\sigma^2}} = -\frac{1}{\sigma^2} \frac{d\sigma^2}{dt} = \frac{2W}{N} \cdot \left\{ 2g \cdot \left(1 - \frac{1}{M^{*2}} \right) - g^2 (M + U) \right\}$$

- But: Momentum distribution $\Psi(\sigma, t)$ changes during cooling
- Therefore: For a complete description of momentum cooling a **Fokker-Planck Equation** for the time evolution of the beam momentum distribution $\Psi(\sigma, t)$ must be solved:

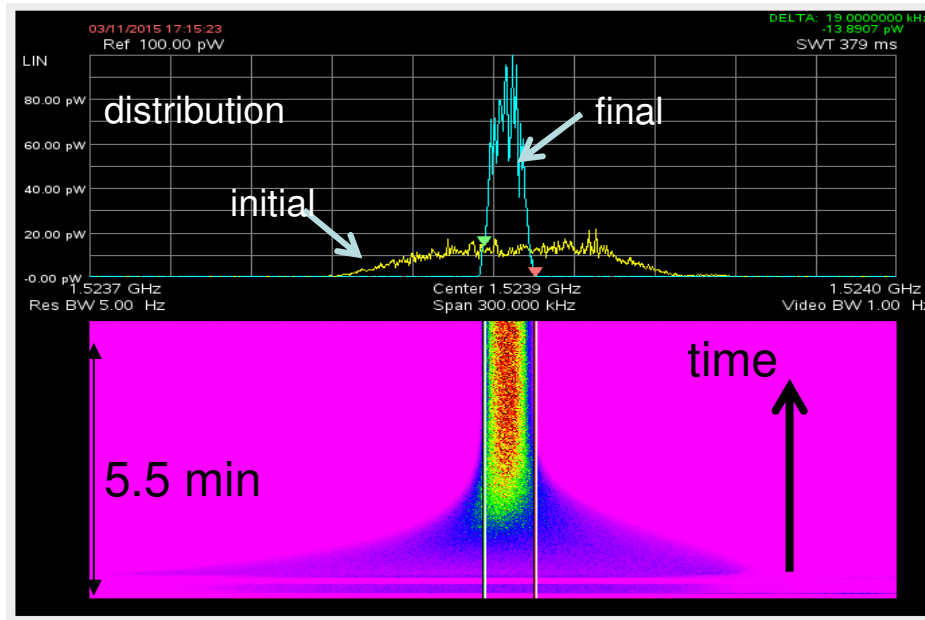
$$\frac{\partial}{\partial t} \Psi(\sigma, t) = -\frac{\partial}{\partial \sigma} \left[F(\sigma, t) \Psi(\sigma, t) - D(\sigma, t) \frac{\partial}{\partial \sigma} \Psi(\sigma, t) \right]$$

Drift: coherent cooling

Diffusion: incoherent heating

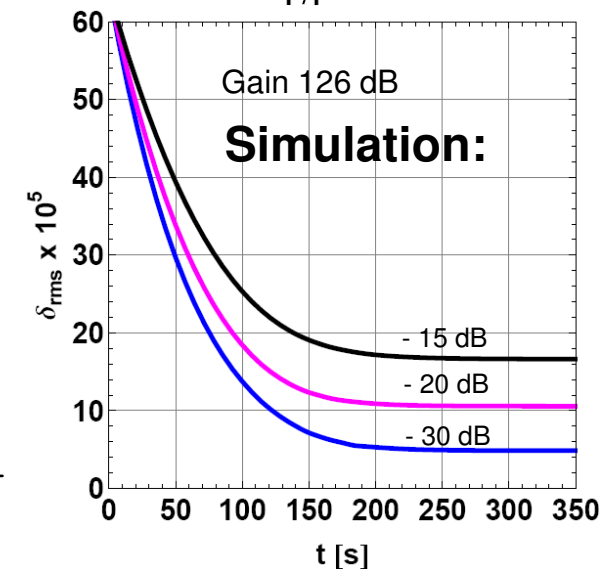
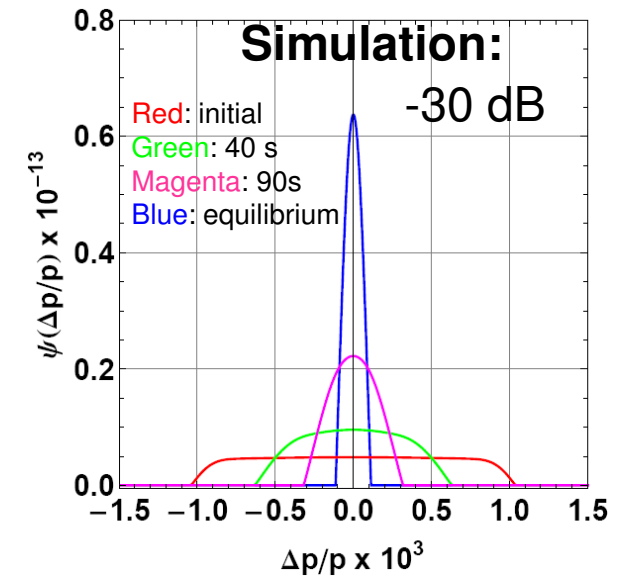
Momentum Cooling at COSY

Measurement:



- Protons $p = 2.425 \text{ GeV/c}$ $\gamma = 2.771$
- Number of protons $N = (6 - 8) \cdot 10^8$
- Lattice: $\gamma_{tr} = 2.343$ measured $\eta = -0.05$
- Dispersion $D = D' = 0$ in telescopes
- Cooling system: band II (1.8 – 3) GHz
- Filter momentum cooling

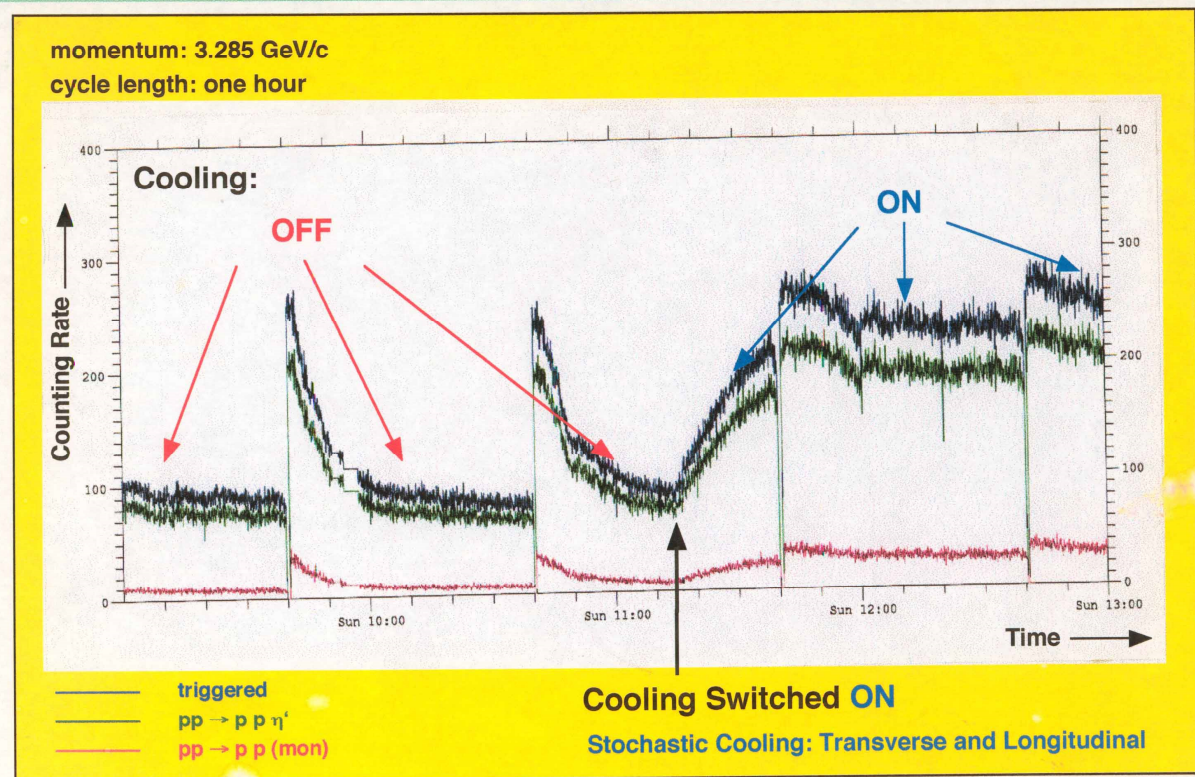
$$\frac{\Delta f}{f_0} = \eta \cdot \frac{\Delta p}{p_0}$$



Stochastic Cooling in the COSY-11 Experiment (September 1998)

- Transverse and Longitudinal Cooling switched ON
- Highest momentum 3.3 GeV/c
- Cycle length one hour
- Counting rate becomes nearly constant when cooling is ON

COSY-11 Counting Rates with and without Stochastic Cooling



COSY11countingrates.cdrow, H. Stockhorst, 4.9.98

Summary (1)

- Stochastic particle beam cooling has been discussed in time domain.
- The finite bandwidth of the signal processing system is too small to resolve a single particle. Instead samples of the continuous coasting beam (DC beam) with millions of particles are resolved.
- This led to the sampling picture.
- Considering a “test” particle in the sample it receives not only a correction at the kicker due to its own error but also “kicks” from the other sample particles.
- This led heuristically to the concept of mixing and heating.
- Unwanted mixing from pickup to kicker that diminishes the coherent cooling effect has been introduced to account for the finite pulse length at the kicker.

Summary (2)

- The application of the sample relations to the beam samples led to the rate equation for stochastic cooling.
- The equation has been improved by unwanted mixing and wanted mixing. Electronic noise that increases the heating term has been introduced ad hoc. For betatron cooling the coherent term is modified so as to include the case of a non-optimal betatron phase advance from pickup to kicker.
- The cooling rate is proportional to the system bandwidth W and inversely proportional to the number of particles N in the beam.
- Schottky particle and electronic noise lead to an equilibrium value in either momentum or betatron cooling. Cryogenic cooling the pickup electrodes and low noise amplifier will reduce the achievable equilibrium value. In addition, the noise-to-signal ratio U can be reduced with high sensitive pickups.

Exercises

- Proof the sample relations.
- Verify the steps in the application of the sample relations to cooling.
- Continue the cooling sampling tables:
 - *Show that the “beam” variance decreases exponentially.*
- Verify the formula for the optimal cooling rate. Determine the optimal gain.
- Discuss why the optimal mixing factor is $M = 1$.
(Note: from the analysis in frequency domain it follows that always $M \geq 1$)
- Discuss why the mixing factor $M > 1$ increases the heating term.
- Discuss the unwanted mixing from PU to KI.
Compare the parabolic kicker pulse with the expression $\frac{I}{W} \int_{f_-}^{f_+} \cos(2\pi f \Delta T_{PK}) df$
derived in frequency domain ($W = f_+ - f_-$).
- Verify the asymptotic behavior of the cooling time.
- The cooling rate was derived under the assumption of a continuous coasting beam (DC beam). Discuss the effect on the cooling rate if the beam gets bunched by an RF cavity. Hint: the bunch length is less than the ring length.

Thanks to the audience for listening